Boolean Agebra

Opening up a computer, a terminal, or practically any other "computerized" item reveals boards containing little black rectangles. These little black rectangles are the integrated circuit (IC) chips that perform the logic of the computer. Each IC can be represented by a Boolean Algebra equation, and vice versa. The representation that is used depends on the context. Boolean Algebra provides a convenient representation and notation for simplifying and solving equations. Digital Electronics provides a layout that can then be implemented with IC chips.

The operators used in these categories are listed in the description of the Digital Electronics category. The logic gates are usually used in Digital Electronics questions; the algebraic equations, symbols and truth tables, in Boolean Algebra. Of course, it is crucial to be able to translate between a digital electronics circuit and its Boolean Algebra notation. The order of operator precedence is NOT; AND and NAND; XOR and EQUIV; OR and NOR. Binary operators with the same level of precedence are evaluated from left to right.

Boolean Algebra Identities

1. $A+B=B+A$ $\quad A*B=B*A$ (Communicative Property)
2. $A+(B+C)=(A+B)+C$ $\quad A*(B*C)=(A*B)*C$ (Associative Property)
3. $A*(B+C)=A*B+A*C$ (Distributive Property)
4. $\overline{A+B}=\overline{A}*\overline{B}$ (DeMorgan's Law)
5. $\overline{A*B}=\overline{A}+\overline{B}$ (DeMorgan's Law)

6. $A + 0 = A$ $A * 0 = 0$

7. $A + 1 = 1$ $A * 1 = A$

8. $A + \overline{A} = 1$ $A * \overline{A} = 0$

9. $A + A = A$ $A * A = A$

10. $\overline{\overline{A}} = A$

11. $A + \overline{A} * B = A + B$

= A * (1+B) + ~A * B

= A + A * B + ~A * B

= A + B * (A + ~A)

= A + B

12. $(A + B) * (A + C) = A + B * C$

13. $(A + B) * (C + D) = A * C + A * D + B * C + B * D$

14. $A * (A + B) = A$

= A * A + A * B

= A + (A * B)

= (A * 1) + (A * B)

= A * (1 + B)

= A * 1 = A

15. *A + (A * B) = A*

16. $A \oplus B = A * \overline{B} + \overline{A} * B$

17. $\overline{A \oplus B} = \overline{A} \oplus B = A \oplus \overline{B}$

Simplify the following expression as much as possible

$\overline{A+B}$

$\overline{AB}$

$\overline{A(A+B)}$

$\overline{B*\overline{A}}$

$A(\overline{B}+A)$

$AC+A$

$AC+ACB$

$(A+B)C+(A+B)$

$A*B*\overline{C}+B*\overline{C}$

$A*(B*\overline{C}+\overline{B}*C)$

$A*(B+\overline{C})$

$\overline{A}B+\overline{A}AB$

$\overline{A}B+1$

$(\overline{A}+A)*CB$

$\overline{A}+1+B+C$

$\overline{\overline{A}B}+\overline{B+C}$

$A(\overline{B}+A)$

$A+AC+\overline{B}+A\overline{B}$

$A(\overline{B}+1)$

$A(1+C)+\overline{B}(1+A+\overline{C})$

$(A*1)+(A*\overline{C})$

Simplify the following expression as much as possible: $\overline{\overline{A(A+B)} + B\overline{A}}$	Using various elementary identities, the expression simplifies as follows: $\overline{\overline{A(A+B)} + B\overline{A}} = \overline{\overline{A(A+B)}} * \overline{B * \overline{A}}$ $= A(A+B) * (\overline{B} + A)$ $= (A)(\overline{B} + A)$ $= A$
Find all ordered pairs (A, B) that make the following expression TRUE. $\overline{\overline{A+B} + \overline{A} * B}$	$\overline{\overline{A+B} + \overline{A} * B} = (\overline{\overline{A+B}})(\overline{\overline{A}B})$ $= (A+B)(A+\overline{B})$ $= AA + A\overline{B} + BA + B\overline{B}$ $= A + A(B+\overline{B}) + 0$ $= A + A(1) = A + A = A$ This yields the solutions (1, 0) and (1, 1).
Simplify the following expression to one that uses only two operators. $(\overline{\overline{A}+\overline{B}} * \overline{C}) + (A * \overline{(B+\overline{C})})$	The evaluation is as follows: $(\overline{\overline{A}+\overline{B}} * \overline{C}) + (A * \overline{(B+\overline{C})})$ $= (A * B * \overline{C}) + (A * \overline{B} * C)$ $= A * (B * \overline{C} + \overline{B} * C)$ $= A * (B \oplus C)$

How many ordered pairs make the following Boolean expression TRUE?

$$\overline{A}(B + A\overline{B}) + \overline{AB}$$

(0,1) (1,0) (0,0)

Simplify the following Boolean algebra expression:

$$\overline{\overline{A}B} + \overline{B + C} + A(\overline{B} + C)$$

$A + \overline{B}$

Simplify the following Boolean expression:

$$A(\overline{B}C) + (\overline{\overline{A} + B})$$

$A\overline{B}$

Which of the following Boolean Algebra expressions are equivalent?

a) $\overline{A\overline{B}} + \overline{B\overline{C}}$ b) $\overline{A} + C$ c) $\overline{\overline{A}C}$ d) 1

a, d

Simplify the following Boolean expression:

$$(A\overline{B} + AC + B\overline{C})(B\overline{C} + \overline{A}B + \overline{A}C)$$

$B\overline{C}$

How many ordered triples make this Boolean expression FALSE?

$$\overline{AB}(\overline{\overline{A} + B + C})(\overline{A + B\overline{C}})$$