Computer Number Systems

All computers – from large mainframes to hand-held micros – ultimately can do one thing: detect whether an electrical signal is "on" or "off". Computer programs in BASIC and Pascal are converted by various pieces of systems software into sequences of bits (*Bi*nary dig*IT*s) which correspond to sequences of on/off (equivalently TRUE/FALSE or 1/0) signals. Proficiency in the binary number system is essential to understanding how a computer works.

Since binary numbers representing moderate values quickly become rather lengthy, bases eight (octal) and sixteen (hexadecimal) are frequently used as short-hand. Octal numbers group binary numbers in bunches of 3 digits and convert the triplet to a single digit between 0 and 7, inclusive. For example, 1001010110_2 = 001 001 010 110_2 = 1126_8. Hexadecimal numbers group binary numbers by fours, and convert the quadruplet to a single digit in the range 0, 1, 2 …, 9, A, B, C, D, E, F. For example, 10110110100101_2 = 0010 1101 1010 0101_2 = $2DA5_{16}$.

Decimal Number: 10 is 10

(1digit) 0 1 2 3 4 5 6 7 8 9

(2 digits) 10 11 12……20……70……80……99

(3 digits) 100 ……500……999

(4 digits) 1000

Binary Number: 2 is 10

(1digit) 0 1

(2 digits) 10 11

(3 digits) 100 101 110 111

(4 digits) 1000

Octal Number: 8 is 10

(1digit) 0 1 2 3 4 5 6 7

(2 digits) 10 11 12 13 47……67……77

(3 digits) 100 ……777

(4 digits) 1000

Hex Number: 16 is 10

(1digit) 0 1 2 3 4 5 6 7 8 9 A B C D E F

(2 digits) 10 11 12 13 14 15 16 17 18 19 1A 1B 1C 1D 1E 1F

20…… FF

(3 digits) 100……FFF

(4 digits) 1000

Decimal to Binary

64	32	16	8	4	2	1
2^6	2^5	2^4	2^3	2^2	2^1	2^0

$8_{10} \rightarrow 1000_2$

			1	0	0	0
64	32	16	8	4	2	1

$25_{10} \rightarrow 11001_2$

		1	1	0	0	1
64	32	16	8	4	2	1

$127_{10} \rightarrow 1111111_2$

1	1	1	1	1	1	1
64	32	16	8	4	2	1

56_{10}

85_{10}

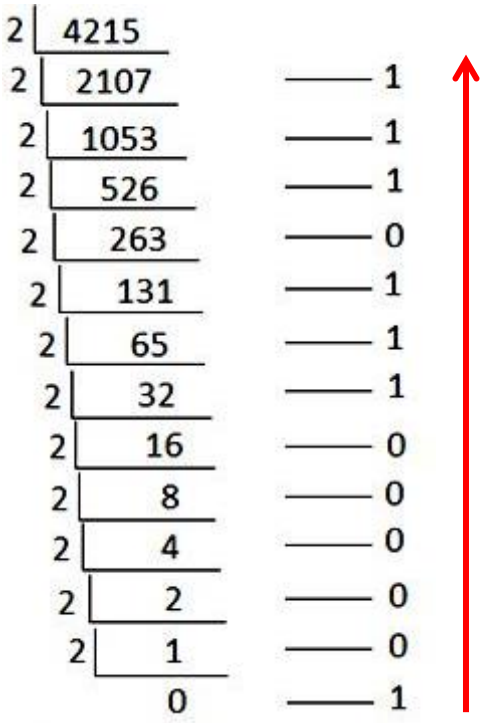
$111_{10} \rightarrow$

$48_{10} \rightarrow$

$72_{10} \rightarrow$

$66_{10} \rightarrow$

4215_{10} → 1000001110111_2



414_{10} →

178_{10} →

200_{10} →

218_{10} →

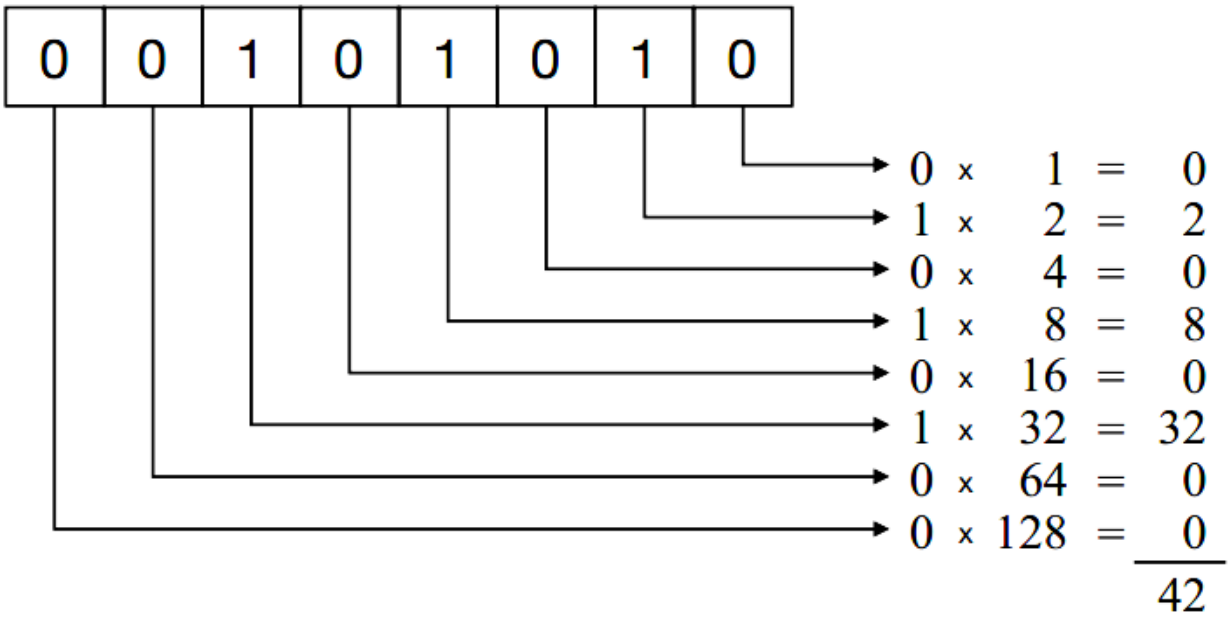
375_{10} →

75_{10} →

87_{10} →

230_{10} →

Binary to Decimal



$1000_2 \rightarrow 8_{10}$

64	32	16	8	4	2	1
			1	0	0	0

1011_2 → 8 * 1 + 4 * 0 + 2 * 1 + 1 * 1 → 11_{10}

11110_2 →

110101_2 →

1011_2 →

10_2 →

111_2 →

11_2 →

Decimal to Octal

64	8	1
8^2	8^1	8^0

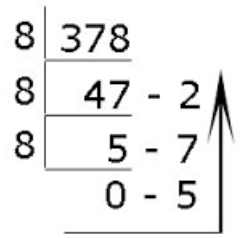
$65_{10} \rightarrow 101_8$

1	0	1
64	8	1

$82_{10} \rightarrow 122_8$

1	2	2
64	8	1

378 ?



$414_{10} \rightarrow$

$178_{10} \rightarrow$

$200_{10} \rightarrow$

$218_{10} \rightarrow$

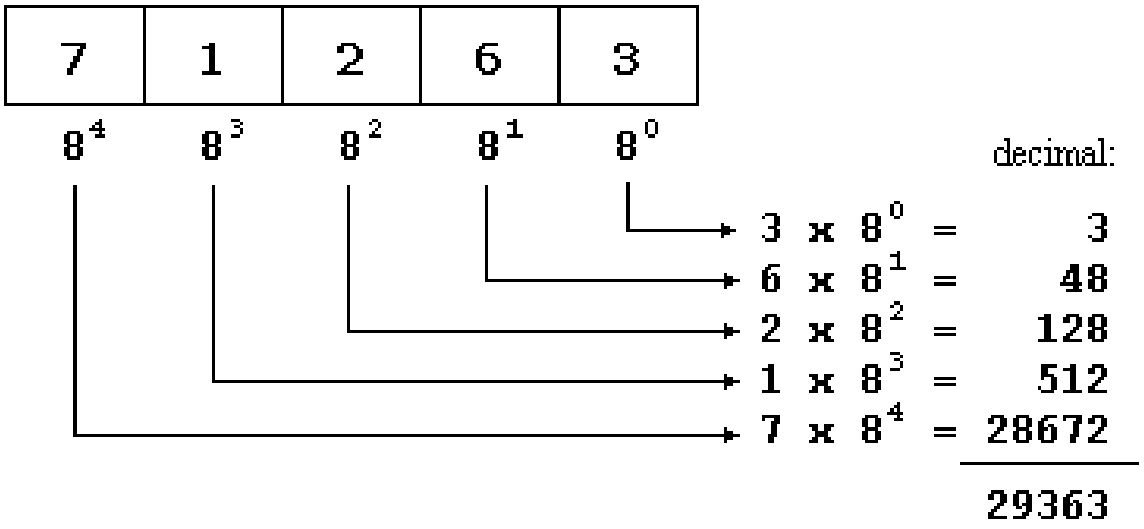
$375_{10} \rightarrow$

$75_{10} \rightarrow$

$87_{10} \rightarrow$

$230_{10} \rightarrow$

Octal to Decimal



111_8 → 64 * 1 + 8 * 1 + 1 * 1 → 64 + 8 + 1 → 73_{10}

47_8 → 4 * 8 + 7 * 1 → 32 + 7 → 39_{10}

55_8 →

77_8 →

126_8 →

210_8 →

26_8 →

222_8 →

Decimal to Hex

414_{10} →

178_{10} →

200_{10} →

218_{10} →

375_{10} →

75_{10} →

87_{10} →

230_{10} →

Hex to Decimal

$3A_{16}$ → 3 * 16 + 10 * 1 → 48 + 10 → 58

$6C1_{16}$ →

$A0C_{16}$ →

14_{16} →

ED_{16} →

11_{16} →

18_{16} →

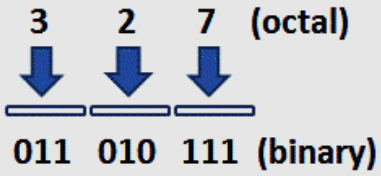
$3D_{16}$ →

BB_{16} →

111_{16} →

Octal to Binary: Split to three digits

To convert octal numbers into binary ones, you only have to convert each digit into 3-bit groups:



33_8 → 011 011_2

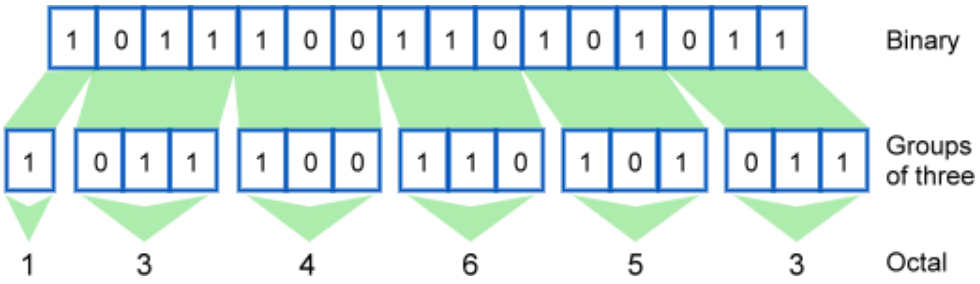
647_8 →

555_8 →

1245_8 →

726_8 →

Binary to Octal: Combine of three digits from right to left



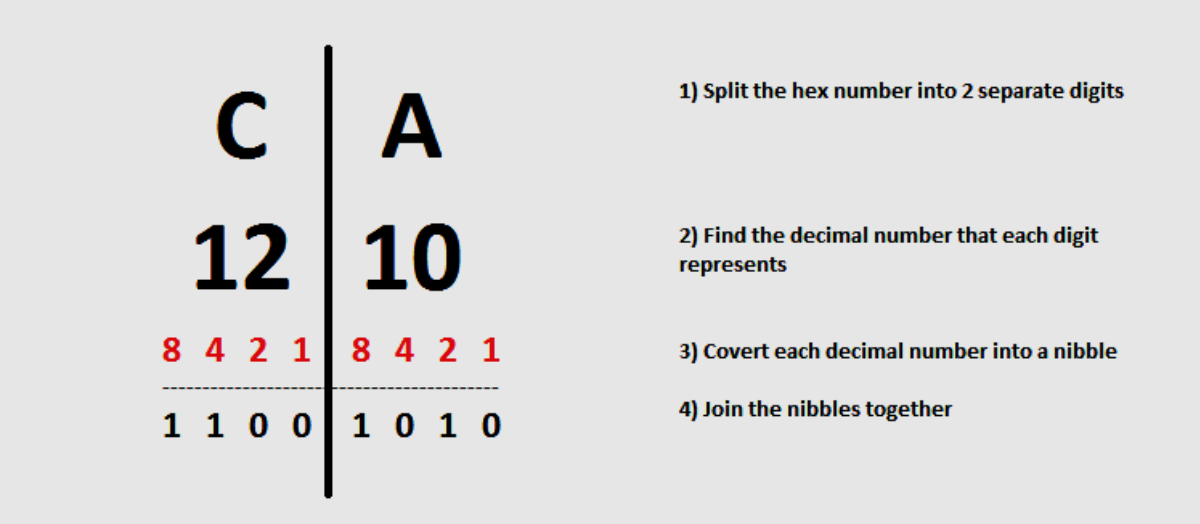
1010110011_2 → 1 010 110 011_2 → 1263_8

1111010_2 →

11010101010101010_2 →

1011111000110_2 →

Hex to Binary: Split to four digits



$3A_{16}$ → 0011 1010_2

$6CD_{16}$ →

ABC_{16} →

$145F_{16}$ →

$FFFED_{16}$ →

333_{16} →

647_{16} →

525_{16} →

$FABC45_{16}$ →

726_{16} →

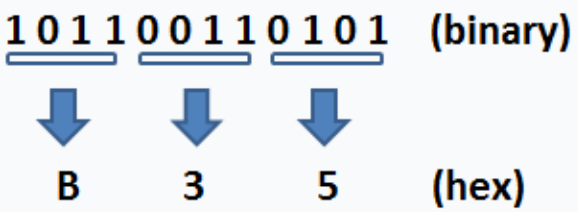
$FABC45_{16}$ →

6485_{16} →

8886_{16} →

Binary to Hex: Combine of four digits from right to left

To convert binary numbers into hexadecimals, you only have to make 4-bit groups and convert directly each group:



1010110011_2 → 10 1011 0011_2 → 2B3

1111010_2 →

110101010101010_2 →

1011111000110_2 →

1101111111110101010_2 →

1011111000110_2 →

1111111111010_2 →

101010101010110_2 →

11000000001110101010_2 →

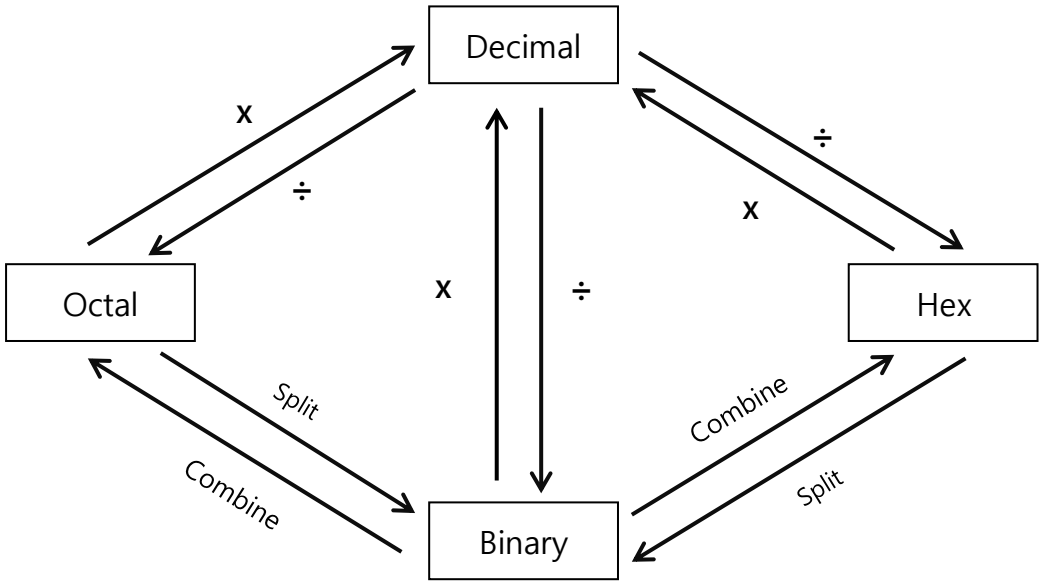
1011000110010_2 →

110000010000010_2 →

101111111110_2 →

1100110010101010_2 →

101110000000110_2 →



Hex to Octal

$3A_{16}$ → 0011 1010 → 00 111 010 → 72_8

$6CD_{16}$

ABC_{16}

$145F_{16}$

$FFFED_{16}$

Octal to Hex

33_8 → 011 011 → 01 1011 → $1B_{16}$

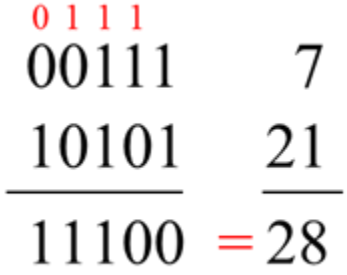
647_8

555_8

1245_8

726_8

Adding in Binary: Attention! 2 is 10 in Binary number



$111000_2 + 111_2 =$

$1111_2 + 1111_2 =$

$1100_2 + 1011_2 =$

$1011001_2 + 110011111_2 =$

$1100_2 + 110111_2 =$

$1111_2 + 1111_2 =$

$1100_2 + 1001_2 =$

$1011000_2 + 110011000_2 =$

$1111_2 + 1101_2 =$

$111111_2 + 1011_2 =$

$101001_2 + 110011_2 =$

$1111_2 + 1000_2 =$

$110011_2 + 1111_2 =$

$1011001_2 + 111111111_2 =$

$1111_2 + 1111111_2 =$

$11001_2 + 100011_2 =$

Adding in Octal: Attention! 8 is 10 in Octal number

$456_8 + 123_8 = 601_8$

1 1		carry
4 5 6	$= 302_{10}$	
+1 2 3	$= 83_{10}$	
6 0 1	$= 385_{10}$	

$65_8 + 521_8 =$

$123_8 + 456_8 =$

$12_8 + 71_8 =$

$105_8 + 756_8 =$

$665_8 + 1025_8 =$

$342_8 + 25_8 =$

$651_8 + 210_8 =$

$34_8 + 76_8 =$

$45_8 + 6_8 =$

$1354_8 + 6246_8 =$

$64251_8 + 254632_8 =$

$3251_8 + 6245_8 =$

Adding in Hex: Attention! 16 is 10 in Hex number

$4A6_{16} + 1B3_{16} = 659_{16}$

1	carry
4 A 6	$= 1190_{10}$
+ 1 B 3	$= 435_{10}$
6 5 9	$= 1625_{10}$

$65AB19_{16} + 2451_{16} =$

$65_{16} + 25C_{16} =$

$12E_{16} + 65FF_{16} =$

$684_{16} + 215_{16} =$

$242BC_{16} + 265_{16} =$

$2138A_{16} + 67B8_{16} =$

$6498CC_{16} + 214F0_{16} =$

$1254_{16} + 51ADE_{16} =$

$356_{16} + 684FA_{16} =$

$3489_{16} + 1025CA_{16} =$

$37D94A_{16} + 6F58_{16} =$

Subtracting in Octal: Attention! 8 is 10 in Octal number

$456_8 - 173_8 = 333_8$

8	borrow
$^{3}456$	$= 302_{10}$
-173	$= 123_{10}$
263	$= 179_{10}$

$521_8 - 65_8 =$

$523_8 - 456_8 =$

$72_8 - 17_8 =$

$605_8 - 56_8 =$

$1665_8 - 77_8 =$

$342_8 - 25_8 =$

$651_8 - 217_8 =$

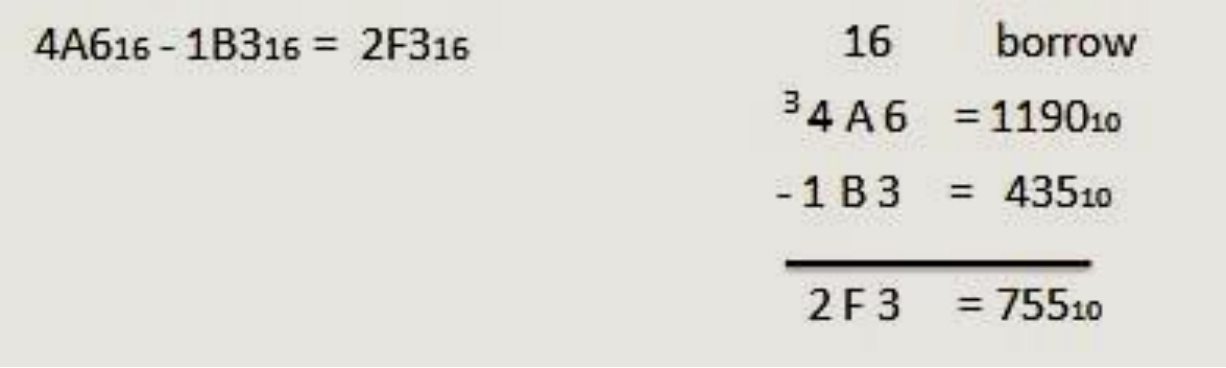
$134_8 - 76_8 =$

$45_8 - 6_8 =$

$11354_8 - 6246_8 =$

$114_8 - 46_8 =$

Subtracting in Hex: Attention! 16 is 10 in Hex number



$65AB19_{16}$ - 2451_{16} =

$AB65_{16}$ - $25C_{16}$ =

$1112E_{16}$ - $65FF_{16}$ =

$F684_{16}$ - 215_{16} =

$242BC_{16}$ - 265_{16} =

$D2138A_{16}$ - $67B8_{16}$ =

$6498CC_{16}$ - $214F0_{16}$ =

$C12D54_{16}$ - $51ADE_{16}$ =

$351F6_{16}$ - $684FA_{16}$ =

$34AB89_{16}$ - $1025CA_{16}$ =

$37D94A_{16}$ - $6F58_{16}$ =

$AF46C_{16}$ - $689A_{16}$ =

Subtracting in Binary: Attention! 2 is 10 in Binary number

$111000_2 - 111_2 = 110001_2$

$1111_2 - 11_2 = 1100_2$

$1100_2 - 1011_2 = 1_2$

$1011001_2 - 11001_2 =$

$1100000_2 - 110111_2 =$

$1111_2 - 1111_2 =$

$1100_2 - 1001_2 =$

$1011000_2 - 110011_2 =$

$1111_2 - 1101_2 =$

$111111_2 - 1011_2 =$

$101001_2 - 11001_2 =$

$1111_2 - 1000_2 =$

$110011_2 - 1111_2 =$

$1011001_2 - 111111_2 =$

$1111_2 - 111_2 =$

$1100100_2 - 100011_2 =$

Solve for X

$X_{8} = FEED_{16} - 6ACE_{16}$

$X_{8} = 56AD_{16} - 3281_{16}$

$X_{8} = F904_{16} - 6ACE_{16}$

$X_{16} = 603_{8} - 376_{8}$

$X_{16} = 120_{8} - 32_{8}$

$X_{16} = 456_{8} - 42_{8}$

$X_{16} = 23_{8} + A3_{16}$

$X_{16} = 25_{8} + F2_{16}$

$X_{16} = 36_{8} + 12_{16}$

$X_{8} = 65_{8} + A3_{16}$

$X_{8} = 77_{8} + A3_{16}$

$X_{8} = 23_{8} + AA_{16}$

$X_{2} = 23_{8} + A_{16}$

$X_{10} = 631_{8} + 83_{16}$

$X_{2} = 777_{8} + A3_{16}$

$X_{8} = 213_{8} + AA_{16}$

$X_{10} = 2113_{8} + BA_{16}$

$X_{10} = 603_{8} + 8A3_{16}$

$X_{8} = 163_{8} + 81B_{16}$

Evaluate the expression and express the final answer in hex.

$10_2 * 61_{16} + 1001_2 * (1011_2 - A_{16})$ CB_{16}

What is the next term in the following sequence in base 10? Express the answer in octal.

1_8, A_{16}, 144_8, $3E8_{16}$ 23420_8

Which has the most 1's in its binary representation?

414_8 $1B5_{16}$ 178_{10} 200_{16} 600_8 $1B5_{16}$

Determine the number of 1's in the binary representation of the solution of the following expression:

$(743_8 - AF_{16} + 110100101000_2) * 256_{10}$ 7

What is the mean (average) of the following numbers expressed in octal?

101010_2 132_8 33_{16} 1010100_2 44_{16} 103_8

Simplify the following and express the answer as a hexadecimal number.

$(23_8 + A3_{16})/2_{10}$ $5B_{16}$

Which expression evaluates to the larger number? Express the answer as a hexadecimal number.

$15_8 + B1C_{16} - 100110_2$ or $AB8_{16} + 127_8 + 1000_2$ $B17_{16}$

Solve for X_{16}:

$2_{10} * X_{16} - 464_8 = 11101110_2 - X_{16}$ $B6_{16}$

Which of the following has the most 1's in its binary representation?

178_{16} 567_8 101110110_2 565_8 377_{10} 567_8

Solve for X_{16}:

$1011011_2 + 2 * X_{16} = 7321_8$ $73B_{16}$