

Classroom Division Solutions

1. Recursive Functions

$$f(1) = 4, \quad f(2) = 4 + 2 = 6, \quad f(3) = 6 + 2 = 8,$$

$$f(4) = 8 + 2 = 10, \dots$$

Arithmetic sequence formed is: 4, 6, 8, 10, 12, ...

So $f(20) = 4 + 19 \cdot 2 = 42$ since 2 is being added 19 times.

1. 42

2. Recursive Functions

$$f(3) = f(2 \cdot 3 - 1) + 1 = f(5) + 1 = 21 + 1 = 22$$

$$f(5) = f(2 \cdot 5 - 1) + 1 = f(9) + 1 = 20 + 1 = 21$$

$$f(9) = f(2 \cdot 9 - 1) + 1 = f(17) + 1 = 19 + 1 = 20$$

$$f(17) = 17 + 2 = 19$$

Now substitute backwards.

2. 22

3. Computer Number Systems

Method I: $2016_{16} = 2 \cdot 16^3 + 0 \cdot 16^2 + 1 \cdot 16 + 6 = 8214$

Keep dividing 8214 by 8 and read the remainders in reverse yields 20026_8

Method II: Write in binary and then regroup in groups of 3.

$$2016_{16} = 0010\ 0000\ 0001\ 0110_2$$

$$= 0\ 010\ 000\ 000\ 010\ 110_2 = 20026_8$$

3. 20026 or 20026_8 **4. Computer Number Systems**

$$327_8 = 3 \cdot 8^2 + 2 \cdot 8 + 7 = 215; \quad 11011011_2 = 128 + 64 + 16 + 8 + 2 + 1 = 219$$

$$11011010_2 = 128 + 64 + 16 + 8 + 2 = 218; \quad 334_8 = 3 \cdot 64 + 3 \cdot 8 + 4 = 220$$

$$330_8 = 3 \cdot 8^2 + 3 \cdot 8 = 216; \quad 11011101_2 = 128 + 64 + 16 + 8 + 4 + 1 = 221$$

$$11011001_2 = 128 + 64 + 16 + 8 + 1 = 217; \quad 332_8 = 3 \cdot 8^2 + 3 \cdot 8 + 2 = 218$$

$$a = 434 \quad b = 438 \quad c = 437 \quad d = 435$$

4. b

5. What Does This Program Do?

The table contains the values of a, b, c, and d after each line.

a	b	c	d
42	20	2	1
22	20	2	1
22	20	4	1
22	20	4	5
22	4	4	5
11	4	4	5

$$2 * (a - 3) / b - b ^ d / c ^ 3 = 2 * (11 - 3) / 4 - 4 ^ 5 / 4 ^ 3$$

$$= 2 * 8 / 4 - 1024 / 64 = 4 - 16 = -12$$

5. -12

ACSL

2016 - 2017

American Computer Science League

Contest #1

Classroom Division Solutions

6. Recursive Functions

$$\begin{aligned}
 f(1) &= 2 \\
 f(2) &= 2 * f(1) + 3 = 2 * 2 + 3 = 7 \\
 f(3) &= 2 * f(2) + 3 = 2 * 7 + 3 = 17 \\
 f(4) &= 2 * f(3) + 3 = 2 * 17 + 3 = 37 \\
 f(5) &= 2 * f(4) + 3 = 2 * 37 + 3 = 77 \\
 f(6) &= 2 * f(5) + 3 = 2 * 77 + 3 = 157
 \end{aligned}$$

6. 157

7. Recursive Functions

$$\begin{aligned}
 f(16) &= 2 * f(13) + 4 = 2 * 0 + 4 = 4 \\
 f(13) &= 2 * f(10) + 4 = 2 * (-2) + 4 = 0 \\
 f(10) &= 2 * f(7) + 4 = 2 * (-3) + 4 = -2 \\
 f(7) &= f([7/2]) - 1 = f(3) - 1 = -2 - 1 = -3 \\
 f(3) &= f([3/2]) - 1 = f(1) - 1 = -1 - 1 = -2 \\
 f(1) &= f([1/2]) - 1 = f(0) - 1 = 0 - 1 = -1 \\
 f(0) &= 0 * 0 - 0 = 0 \quad \text{Now substitute backwards.}
 \end{aligned}$$

7. 4

8. Computer Number Systems

$$\begin{aligned}
 10_2 &= 2 & \text{So } 10_2 * 61_{16} + 1001_2 * (1011_2 - A_{16}) \\
 61_{16} &= 6 * 16 + 1 = 97 & = 2 * 97 + 9 * (11 - 10) \\
 1001_2 &= 1 * 8 + 0 * 4 + 0 * 2 + 1 = 9 & = 194 + 9 * 1 = 194 + 9 \\
 1011_2 &= 1 * 8 + 0 * 4 + 1 * 2 + 1 = 11 & = 203 = CB_{16} \\
 A_{16} &= 10
 \end{aligned}$$

8. CB₁₆ or CB

9. Computer Number Systems

$$\begin{aligned}
 1_8 &= 1 \\
 A_{16} &= 10 \\
 144_8 &= 100 \\
 3E8_{16} &= 1000 \\
 \text{So the next term is } 10000 &= 23420_8
 \end{aligned}$$

9. 23420₈ or 23420

10. What Does This Program Do?

The table contains the values of a, b, c, d and e after each line.

a	b	c	d	e
24	5	2	4	1
6	5	2	4	1
6	5	7	4	1
6	5	7	5	1
6	1	7	5	1
12	1	7	5	1
10	1	7	5	1

$$\begin{aligned}
 \text{So } a + c / b - d * (a / 5 + d / b) / c - c / (b + d + e) \\
 = 10 + 7 / 1 - 5 * (10 / 5 + 5 / 1) / 7 - 7 / (1 + 5 + 1) = 11
 \end{aligned}$$

10. 11