Graph Theory Solutions

1. **02-03 C3 Graph Theory**

A cycle is a simple path with no vertices repeated except for the first and last vertex point. The following cycles exist: 9
ABA, ADA, ABDA, ABEA, ADCBA, BDCB, BEDCB, ABEDA, ADCBEA

2. 02-03 C3 Graph Theory

The adjacency matrix is: 15

	A	**B**	**C**	**D**	**E**
A	**0**	**1**	**0**	**1**	**0**
B	**1**	**0**	**0**	**1**	**1**
C	**0**	**1**	**0**	**0**	**0**
D	**1**	**0**	**1**	**0**	**0**
E	**1**	**0**	**0**	**1**	**0**

3. **03-04 C3 Graph Theory**

The adjacency matrix squared is: 2

	A	B	C	D						
A	0	1	0	0	2		0	0	1	1
B	0	0	1	1		=	1	1	1	0
C	1	1	0	0			0	1	1	1
D	0	0	1	0			1	1	0	0

The number of paths from A is found by summing the top row of the squared matrix.

4. **03-04 C3 Graph Theory**

The simple paths are: ABC, ABD, BCD, CBD, BDA, DAB, CDA 7

5. **04-05 C3 Graph Theory**

The adjacency matrix is: As shown

	A	**B**	**C**	**D**
A	0	1	1	0
B	0	1	0	1
C	0	0	0	1
D	1	1	0	0

6. **04-05 C3 Graph Theory**

Squaring the adjacency matrix gives all the paths of length 2 from each vertex. 7

$$\begin{vmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}^2 = \begin{vmatrix} 1 & 2 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

The sum of the elements in the squared matrix is 7

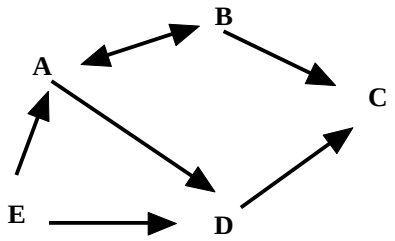
7. **05-06 C3 Graph Theory**

The adjacency matrix is: As shown

0	**1**	**1**	**1**	**0**
0	**1**	**1**	**0**	**0**
1	**1**	**0**	**1**	**0**
0	**1**	**0**	**0**	**1**
1	**0**	**0**	**0**	**0**

8. **05-06 C3 Graph Theory**

The graph is similar to: As shown



9. **06-07 C3 Graph Theory**

The adjacency matrix squared is: 8

$$\begin{vmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{1} & \mathbf{1} & \mathbf{0} \end{vmatrix}^2 = \begin{vmatrix} \mathbf{2} & \mathbf{2} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} \end{vmatrix}$$

Summing up all of the entries yields 8 total paths of length 2.

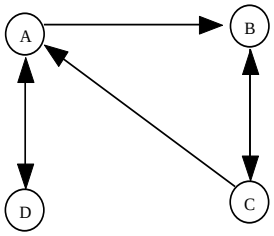
10. **06-07 C3 Graph Theory**

The adjacency matrix is: As shown

$$\begin{vmatrix} \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{vmatrix}$$

11. **07-08 C3 Graph Theory**

Arrows are drawn from the first vertex listed to the second one. There are 4 vertices and 6 edges. — As shown



12. **07-08 C3 Graph Theory**

The adjacency matrix is: — **A**s shown

$$\begin{vmatrix} \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \end{vmatrix}$$

13. **08-09 C3 Graph Theory**

Squaring an adjacency matrix produces all the paths of length 2. Adding the entries gives the number of paths of length 2. — 20

$$\begin{vmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{vmatrix}^2 = \begin{vmatrix} 1 & 1 & 1 & 2 \\ 1 & 2 & 1 & 0 \\ 2 & 2 & 1 & 2 \\ 1 & 1 & 1 & 1 \end{vmatrix}$$

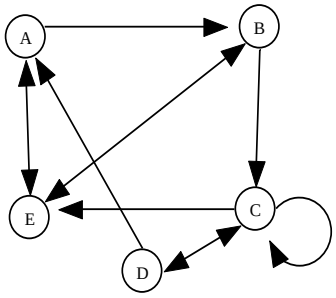
14. **08-09 C3 Graph Theory**

The adjacency matrix is: — As shown

$$\begin{vmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{vmatrix}$$

15. **09-10 C3 Graph Theory**

Arrows are drawn from the first vertex listed to the second one. There are 5 vertices and 11 edges. Adjacency matrix as shown in the answer column.



0	1	0	0	1
0	0	1	0	1
0	0	1	1	1
1	0	1	0	0
1	1	0	0	0

16. **09-10 C3 Graph Theory**

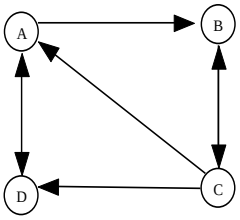
The adjacency matrix squared is: 12

$$\begin{vmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{vmatrix}^2 = \begin{vmatrix} 0 & 1 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 2 & 2 & 0 \end{vmatrix}$$

The sum of all of the entries is 12.

17. **10-11 Graph Theory**

Arrows are drawn from the first vertex listed to the second one. As shown
There are 4 vertices and 7 edges.



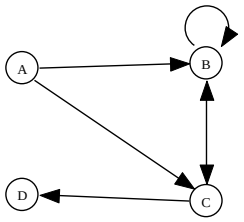
18. **10-11 Graph Theory**

The adjacency matrix is squared B to D has 4 paths of length 2. 4

$$\begin{vmatrix} \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} \end{vmatrix}^{\mathbf{2}} = \begin{vmatrix} \mathbf{2} & \mathbf{3} & \mathbf{1} & \mathbf{3} \\ \mathbf{2} & \mathbf{3} & \mathbf{1} & \mathbf{4} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{2} & \mathbf{1} & \mathbf{2} \end{vmatrix}$$

19. **11-12 C3 Graph Theory**

The graph is shown below: As shown



20. **11-12 C3 Graph Theory**

The adjacency matrix is shown below: As shown

1	1	0	1	0
0	0	1	0	0
0	0	0	0	0
1	1	1	0	0
1	0	0	1	0

21. **12-13 C3 Graph Theory**

The adjacency matrix is: As shown

0	**0**	**1**	**1**	**1**
1	**1**	**1**	**0**	**0**
1	**0**	**0**	**1**	**1**
0	**0**	**0**	**0**	**0**
0	**0**	**1**	**1**	**0**

22. **12-13 C3 Graph Theory**

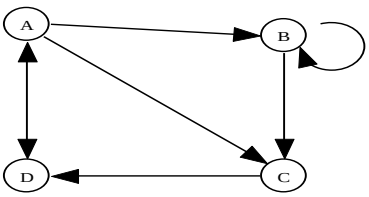
The adjacency matrix squared is: 11

1	1	1	1	2		2	4	2	3
1	1	1	1		=	2	4	2	3
0	1	0	1			1	2	1	1
0	1	0	0			1	1	1	1

Adding the second row results in 11 paths from vertex B.

23. **13-14 C3 Graph Theory**

The graph is similar to this: As shown



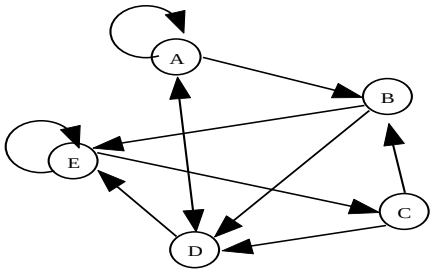
24. **13-14 C3 Graph Theory**

The adjacency matrix is: As shown

$$\begin{vmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \end{vmatrix}$$

25. **14-15 C3 Graph Theory**

The graph must be similar to: As shown



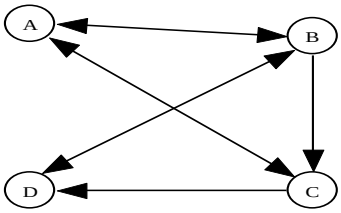
26. **14-15 C3 Graph Theory**

The adjacency matrix is: As shown

$$\begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \end{vmatrix}$$

27. **15-16 C3 Graph Theory**

The graph is similar the one below: As shown



28. **15-16 C3 Graph Theory**

The adjacency matrix is: As shown

$$\begin{vmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{vmatrix}$$