

## Recursive Functions Solutions

### 1. 02-03 C1 Recursive Functions

$$\begin{aligned} F(18) &= F(18 - 4) - 2 = -8 - 2 = -10 \\ F(14) &= F(14 - 4) - 2 = -6 - 2 = -8 \\ F(10) &= F(10 - 4) - 2 = -4 - 2 = -6 \\ F(6) &= -4 \end{aligned} \qquad -10$$

### 2. 02-03 C1 Recursive Functions

$$F(F(F(3))) = F(F(9)) = F(10) = 11 \qquad 11$$

### 3. 03-04 C1 Recursive Functions

$$\begin{aligned} f(10) &= f(8) + 1 = 6 + 1 = 7 \\ f(8) &= f(6) + 1 = 5 + 1 = 6 \\ f(6) &= f(4) + 1 = 4 + 1 = 5 \\ f(4) &= 4 \end{aligned} \qquad 7$$

### 4. 03-04 C1 Recursive Functions

$$\begin{aligned} f(8) &= f(f(5)) + 2 = f(-2) + 2 = -1 + 2 = 1 \\ f(5) &= f(f(2)) + 2 = f(-1) + 2 = -4 + 2 = -2 \\ f(2) &= 2^2 - 5 = -1 \\ f(-1) &= (-1)^2 - 5 = -4 \\ f(-2) &= (-2)^2 - 5 = -1 \end{aligned} \qquad 1$$

### 5. 04-05 C1 Recursive Functions

$$\begin{aligned} f(5) &= f(f(5 - 3)) + 3 \\ &= f(f(2)) + 3 = f(2) + 3 = 2 + 3 = 5 \\ f(2) &= f(f(2 - 3)) + 3 \\ &= f(f(-1)) + 3 = f(-1) + 3 = -1 + 3 = 2 \end{aligned} \qquad 5$$

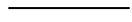
### 6. 04-05 C1 Recursive Functions

$$\begin{aligned} f(13, 2) &= f(10, 4) - 1 = 7 - 1 = 6 \\ f(10, 4) &= f(7, 6) - 1 = 8 - 1 = 7 \\ f(7, 6) &= f(4, 8) - 1 = 9 - 1 = 8 \\ f(4, 8) &= f(1, 10) - 1 = 10 - 1 = 9 \\ f(1, 10) &= f(3, 6) + 1 = 9 + 1 = 10 \\ f(3, 6) &= 9 \end{aligned} \qquad 6$$

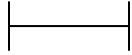
### 7. 05-06 C1 Recursive Functions

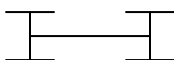
$$\begin{aligned} F(5) &= F(F(4)) - 2 = F(-5) - 2 = 22 - 2 = 20 \\ F(4) &= F(F(3)) - 2 = F(0) - 2 = -3 - 2 = -5 \\ F(3) &= F(1) + 1 = -1 + 1 = 0 \\ F(1) &= F(-1) + 1 = -2 + 1 = -1 \\ F(-1) &= 1 - 3 = -2 \\ F(0) &= 0^2 - 3 = -3 \\ F(-5) &= 25 - 3 = 22 \end{aligned} \qquad 20$$

**8. 05-06 C1 Recursive Functions**

STAGE 1  1 segment added

16

STAGE 2  2 segments added

STAGE 3  4 segments added

Following the pattern, 8 segments are added in stage 4 and 16 in stage 5.

**9. 06-07 C1 Recursive Functions**

$$f(23) = f(17) + 2 = 79$$

79

$$f(17) = f(11) + 2 = 77$$

$$f(11) = f(5) + 2 = 75$$

$$f(5) = 2 * f(6) - 1 = 73$$

$$f(6) = 36 + 1 = 37$$

**10. 06-07 C1 Recursive Functions**

$$f(5, 11) = f(6, 9) + 1 = 6$$

6

$$f(6, 9) = f(7, 7) + 1 = 5$$

$$f(7, 7) = 2 * f(9, 6) - 2 = 4$$

$$f(9, 6) = 3 * 9 - 4 * 6 = 3$$

**11. 07-08 C1 Recursive Functions**

$$f(150) = f(30 - 2) + 10 = 24 \quad \text{So } (f(f(f(f(150)))) \quad 6$$

$$f(28) = f(5 - 2) + 10 = 14 \quad = f(f(f(24)))$$

$$f(3) = 1 + 3 = 4 \quad = f(f(13))$$

$$f(24) = f(4 - 2) + 10 = 13 \quad = f(4)$$

$$f(2) = 1 + 2 = 3 \quad = 6$$

$$f(13) = f(4 + 2) - 5 = 4$$

$$f(6) = 3 + 6 = 9$$

$$f(4) = 2 + 4 = 6$$

**12. 07-08 C1 Recursive Functions**

$$f(10) = f(10 - 3) + 10 = f(7) + 10 = 35 + 10 = 45 \quad 45$$

$$f(7) = f(7 + 1) - 2 = f(8) - 2 = 37 - 2 = 35$$

$$f(8) = f(8 + 1) - 2 = f(9) - 2 = 39 - 2 = 37$$

$$f(9) = f(9 - 3) + 9 = f(6) + 9 = 30 + 9 = 39$$

$$f(6) = 6^2 - 6 = 36 - 6 = 30 \quad \text{Now substitute backwards.}$$

13. **08-09 C1 Recursive Functions**

$$\begin{aligned} f(10) &= f(10 - 2) + 10 = f(8) + 10 = 30 + 10 = 40 \\ f(8) &= f(8 - 2) + 8 = f(6) + 8 = 22 + 8 = 30 \\ f(6) &= f(6 - 2) + 6 = f(4) + 6 = 16 + 6 = 22 \\ f(4) &= f(4 + 1) - 4 = f(5) - 4 = 20 - 4 = 16 \\ f(5) &= 4 * 5 = 20 \quad \text{Now substitute backwards.} \end{aligned} \quad 40$$

14. **08-09 C1 Recursive Functions**

# hours	12	24	36	48	60.....	108	120	5
# amoeba	2	4	8	16	32 .....	512	1024	
# days	0.5	1	1.5	2	2.5.....	4.5	5	

15. **09-10 C1 Recursive Functions**

$$\begin{aligned} f(12) &= f(10) + 1 = 4 + 1 = 5 \\ f(10) &= f(8) + 1 = 3 + 1 = 4 \\ f(8) &= f(6) + 1 = 2 + 1 = 3 \\ f(6) &= f(7) - 2 = 4 - 2 = 2 \\ f(7) &= 4 \quad \text{Now substitute backwards.} \end{aligned} \quad 5$$

16. **09-10 C1 Recursive Functions**

$$\begin{aligned} f(10,4) &= f(8,5) - 3 = -10 - 3 = -13 \\ f(8,5) &= f(6,6) - 3 = -7 - 3 = -10 \\ f(6,6) &= f(8,4) + 1 = -8 + 1 = -7 \\ f(8,4) &= f(6,5) - 3 = -5 - 3 = -8 \\ f(6,5) &= f(4,6) - 3 = -2 - 3 = -5 \\ f(4,6) &= 4^2 - 3 * 6 = 16 - 18 = -2 \quad \text{Substitute backwards.} \end{aligned} \quad -13$$

17. **10-11 C1 Recursive Functions**

$$\begin{aligned} f(36) &= f(18) - 3 = 23 - 3 = 20 \\ f(18) &= f(9) - 3 = 26 - 3 = 23 \\ f(9) &= f(10) + 4 = 22 + 4 = 26 \\ f(10) &= f(5) - 3 = 25 - 3 = 22 \\ f(5) &= 25 \quad \text{Now substitute backwards.} \end{aligned} \quad 20$$

18. **10-11 C1 Recursive Functions**

$$\begin{aligned} f(1,11) &= f(3,8) + 2 = 23 + 2 = 25 & f(6,2) &= 2 * 6 + 5 * 2 = 22 \\ f(3,8) &= f(5,5) + 2 = 21 + 2 = 23 \\ f(5,5) &= f(4,5) - 3 = 24 - 3 = 21 \\ f(4,5) &= f(6,2) + 2 = 24 \quad \text{Substitute backwards.} \end{aligned} \quad 25$$

**19. 11-12 C1 Recursive Functions**

$$\begin{aligned} f(12) &= f(10) + 3 = 7 + 3 = 10 && 10 \\ f(10) &= f(8) + 3 = 4 + 3 = 7 \\ f(8) &= f(6) + 3 = 1 + 3 = 4 \\ f(6) &= 6 - 5 = 1 \quad \text{Now substitute backwards.} \end{aligned}$$

**20. 11-12 C1 Recursive Functions**

$$\begin{aligned} f(12, 4) &= f(11, 6) + 1 = 2 && 2 \\ f(11, 6) &= f(10, 8) + 1 = 1 \\ f(10, 8) &= f(9, 10) + 1 = 0 \\ f(9, 10) &= 9 - 10 = -1 \quad \text{Now substitute backwards.} \end{aligned}$$

**21. 02-03 C1 Recursive Functions**

The first time through the function gives 1 painted triangle as shown. 121  
The second recursion gives 3, the third gives 9, the fourth gives 27 and the fifth gives 81.  $1 + 3 + 9 + 27 + 81 = 121$ . This is Sierpinski's Triangle.

**22. 05-06 C1 Recursive Functions**

The sequence in length times the number of segments is as follows: 96  
 $16(1) + 8(2) + 4(4) + 2(8) + 1(16) + 0.5(32) = 96$ .

**23. 06-07 C1 Recursive Functions**

In the first step  $3^0 = 1$  triangle is removed. 81  
In the second step  $3^1 = 3$  triangles are removed.  
In the third step  $3^2 = 9$  triangles are removed.  
In this manner in the fifth step  $3^4 = 81$  triangles are removed.